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2004 J. Phys. A: Math. Gen. 37 L47

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LETTER TO THE EDITOR

A new class of short distance universal amplitude ratios

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Received 11 September 2003

Published 9 January 2004

Online at stacks.iop.org/JPhysA/37/L47 (DOI: 10.1088/0305-4470/37/4/L01)

Abstract

We propose a new class of universal amplitude ratios which involve the first terms of the short distance expansion of the correlators of a statistical model in the vicinity of a critical point. We will describe the critical system with a conformal field theory (UV fixed point) perturbed by an appropriate relevant operator. In two dimensions the exact knowledge of the UV fixed point allows for accurate predictions of the ratios and in many nontrivial integrable perturbations they can even be evaluated exactly. In three-dimensional $O(N)$ scalar systems, feasible extensions of some existing results should allow perturbative expansions for the ratios to be obtained. By construction these universal ratios are a perfect tool to explore the short distance properties of the underlying quantum field theory even in regimes where the correlation length and one-point functions are not accessible in experiments or simulations.

PACS numbers: 05.50.+q, 75.10.Hk, 11.10.Kk

1. Introduction

Universal amplitude ratios play a major role in modern statistical mechanics [1–3] of critical systems. Like critical indices these ratios can be used to characterize and identify universality classes. In this respect they are in general more useful than the critical indices since they are usually simpler to measure and can vary by a larger amount, thus allowing a more efficient discrimination among different universality classes.

In this letter we propose a new class of universal ratios related to the short distance expansion of the correlators of a statistical model near criticality: by construction they provide a perfect tool to probe the short distance properties of the underlying quantum field theory

(QFT in the following) even in regimes where the correlation length and one-point functions are not accessible in experiments or simulations.

We shall discuss the construction of these ratios with a general formalism (for a d -dimensional QFT with a generic operator content), but we will be mainly interested in two-dimensional systems where the necessary assumptions are satisfied in many nontrivial cases. In particular we have in mind some integrable perturbations of conformal field theories (CFT) in which case these short distance universal ratios can be evaluated *exactly*.

This letter is organized as follows. In section 2, after a short reminder of known results about QFT and standard amplitude ratios, we shall construct our new universal ratios. Then in section 3 we shall discuss, as a topical example, the case of the magnetic perturbation of the Ising model.

2. Universal ratios

Let us consider a statistical model near criticality, i.e. a d -dimensional QFT with operators ϕ_α^0 , regularized by a length cut-off $a \ll \xi$ (ξ being the correlation length of the system). We will assume that properties of the statistical model in the short distance limit $a \ll r \ll \xi$ are described by a renormalized QFT obtained by an appropriate relevant perturbation $\int d^d x g_i \phi_i$ of a CFT. (We denote generic renormalized operators by ϕ_α , and their scaling dimension by $2\Delta_\alpha$; latin indices i, j, \dots will be reserved to relevant operators, such that $2\Delta_j < d$. As a consequence the coupling g_i of the perturbing operator ϕ_i will have scaling dimension $d - 2\Delta_i$, such that $\xi^{-1} \propto g_i^{1/(d-2\Delta_i)}$.) In general any regularized operator will be equal to an appropriate mixing of renormalized operators with equal or lower scaling dimension, plus irrelevant corrections. The mixing can happen only if appropriate selection rules on scaling dimensions of operators and coupling are satisfied and symmetries of the theory are preserved. *We assume that no mixing occurs among the relevant operators of the QFT* to which we restrict our analysis. In this case we are left with simple multiplicative relations (up to irrelevant corrections): $\phi_j^0 = Z_j^{-1} \phi_j + \dots$ in which the factors Z_j are the only non-universal avatar of the original statistical model. The invariance of the perturbation, $\int g_i^0 \phi_i^0 = \int g_i \phi_i + \dots$, implies the relation $g_i^0 = Z_i g_i$. It is clear that any scale invariant constant built up from relevant operators and coupling that is invariant by the rescaling

$$\phi_j \rightarrow K_j^{-1} \phi_j \quad g_i \rightarrow K_i g_i \quad (1)$$

will not depend on Z_j and as a consequence will be universal. (We borrow from [7] the name ‘metric factors’ for our normalization constants K_j .)

Let us now review some known universal amplitude ratios related to VEV and correlators (for a thorough review and additional references see [2, 3]).

Let us start with ratios involving one-point functions of relevant operators and denote

$$\langle \phi_j \rangle_i = A_j^i g_i^{\frac{2\Delta_j}{d-2\Delta_i}} \quad (2)$$

the vacuum expectation value (VEV) of the operator ϕ_j in the theory obtained perturbing with respect to ϕ_i . Clearly, the scaling equation (1) is equivalent to replacing

$$A_j^i \rightarrow K_j A_j^i (K_i)^{\frac{2\Delta_j}{d-2\Delta_i}} \quad (3)$$

in equation (2). The only remaining task to obtain universal ratios is to construct suitable combinations of the VEV (or of their derivatives w.r.t. g_i) so as to eliminate all the metric factors (see [4–7] for recent applications). An exhaustive list of these combinations can be found, for instance, in [2, 3].

A similar approach can be followed also if we take, instead of one-point functions, the expectation value of two (or more points) functions. Much work has been done in past years in the context of $O(N)$ scalar models: for instance it was soon realized (see [3, 10] and references therein) that the connected spin correlator as a function of r/ξ is a universal function (up to an overall constant) whose asymptotics can be parametrized in terms of universal constants.

When studying connected correlators in the regime $r \gg \xi \gg a$ one has access to universal ratios of the so-called ‘overlap amplitudes’. This line of research has been recently pushed forward in the case of the 2D Ising model perturbed by a magnetic field [8] and for the thermal perturbation of the 3D Ising model [9].

The complementary possibility is to look at the short distance behaviour $a \ll r \ll \xi$ of correlators. In the context of $O(N)$ scalar models Wilson’s operator product expansion (OPE) was revealed to be [11] the appropriate theoretical tool to deal with the short distance regime and $\epsilon = 4 - d$ expansion techniques have been applied to describe correlator asymptotics and compare them with simulations or with experiments (see [3]).

In this letter we propose a new class of universal amplitudes—obtained from ratios of correlators in the short distance regime—that do not require additional measures of ξ or VEV: the idea is that if one is looking at very small perturbations of the critical point it might happen that the correlation length and VEV estimates are not accessible in the particular experimental or numerical realization in which one is interested.

We pursue the description of short distance asymptotics by applying a specialization of the OPE technique to perturbed CFT recently proposed in [12–15] and known as the infrared safe approach (IRS). Roughly speaking, the main ingredient of the IRS approach is simply to choose a renormalization scheme in which the Wilson coefficients of the OPE have a regular, IR safe, perturbative expansion with respect to the coupling and then to expand them in OPE (see [14, 15] for all orders of IRS formulae and many cautionary remarks on their range of validity). The approach can be extended in principle to n -point correlators and irrelevant operators, but in the following we shall concentrate on the two-point functions of relevant operators only, because the hypotheses of non-mixing and of the absence of divergences—see below—are likely to be satisfied in this case and because in more complex generalizations the amplitude ratios will be too difficult to study from a numerical point of view.

We choose as expansion variable the scaling combination $t \equiv g_i r^{d-2\Delta_i} \propto (r/\xi)^{d-2\Delta_i}$ and rescale the correlators as follows:

$$F_{jk}^i(t) \equiv r^{2\Delta_j+2\Delta_k} \langle \phi_j \phi_k \rangle_i. \quad (4)$$

(Note that $\langle \phi_j \phi_k \rangle_i$ is just the expectation value of the two operators and *not* the connected correlator.) In this notation IRS gives

$$F_{jk}^i(t) \sim \sum_{\alpha, n} A_\alpha^i \frac{\partial^n \widehat{C}_{jk}^\alpha}{\partial g_i^n} t^{\frac{2\Delta_\alpha}{d-2\Delta_i} + n} \equiv \sum_a f_{jka}^i t^{z_{jka}^i} \quad (5)$$

where the constants f_{jka}^i are suitable combinations of $\partial^n \widehat{C}_{jk}^\alpha / \partial g_i^n$ (derivatives of Wilson’s coefficients evaluated at $g_i = 0$ and scaled by appropriate powers of r to obtain adimensional quantities) and of the amplitudes A_α^i of operators’ VEV, defined as in equation (2); z_{jka}^i are appropriate combinations of scaling dimensions. Note that in equation (5) we assumed a powerlike short-distance expansion in terms of the variable t (at least up to the order we will need): in general it may happen that the underlying QFT contains pairs of ‘resonant’ operators and terms proportional to powers of $\log t$ may appear in the correlators at some order in the expansion (a typical example of this situation is the thermal perturbation of the 2D Ising

model). If logs appear in the terms of the expansion the present treatment cannot be applied as it is (we shall deal with these peculiar cases in forthcoming publications).

The key point for the construction of universal ratios is that the scaling equation (1) is equivalent to the nontrivial rescaling

$$f_{jka}^i \rightarrow K_j K_k K_i^{z_{jka}^i} f_{jka}^i \quad (6)$$

(mixing factors from $\langle \phi_\alpha \rangle$ are compensated by those which come from the corresponding redefinition of the Wilson coefficients, even in the general case of matricial mixing). All we need at this point is to look for combinations in which the non-universal factors cancel among each other as in the case of standard amplitude ratios. This can be performed in two steps. First by combining the correlators in appropriate ratios to eliminate the trivial overall dependence on K_j, K_k ,⁵ and then by choosing a combination of ratios to have a finite non-zero result in the $t \rightarrow 0$ limit (this step replaces the knowledge of the correlation length in the standard approach).

There are in general several ways to construct these combinations, but it is clear that the most interesting ones are those which involve the smallest number of different operators and thus have greater chances to be observed in experiments or in simulations. In this letter we propose two options which—in our opinion—are the best candidates. Let us look at them in detail.

2.1. QFTs with three or more relevant operators

Let us choose a model (such as for instance the tricritical Ising model) which contains at least three non-trivial relevant operators (it does not matter if one of the three is the same ϕ_i which we use as a perturbation). Let us call them ϕ_1, ϕ_2 and ϕ_3 , and construct the following combinations:

$$R_{12}^i \equiv \frac{\langle \phi_1 \phi_1 \rangle_i \langle \phi_2 \phi_2 \rangle_i}{[\langle \phi_1 \phi_2 \rangle_i]^2} = \frac{F_{11}^i F_{22}^i}{[F_{12}^i]^2} \quad (7)$$

and similarly for the pairs (ϕ_1, ϕ_3) and (ϕ_2, ϕ_3) . The small t expansion of the R_{jk}^i can be reconstructed from that of their components and we can find in general

$$R_{jk}^i \sim r_{jk}^i t^{z_{jk}^i} + \dots \quad (8)$$

where r_{jk}^i and z_{jk}^i are simple combinations of the f_{jka}^i and z_{jka}^i which appear in small t expansions (see equation (5)) of the correlators involved in the ratio.

The reader can easily check that the overall metric factors K_j, K_k disappear in each of the combinations R_{jk}^i . At this point one can construct one or more universal combinations out of these ratios by combining the R_{jk}^i so as to cancel the leading t dependence. The $t \rightarrow 0$ limit of any one of these combinations is clearly an universal quantity and can be exactly expressed in terms of the Wilson coefficients, their derivatives and the one-point functions of the theory.

For instance one such combination could be

$$Q_{123}^i \equiv \lim_{t \rightarrow 0} \frac{(R_{12}^i)^{z_{13}^i}}{(R_{13}^i)^{z_{12}^i}} = \frac{(r_{12}^i)^{z_{13}^i}}{(r_{13}^i)^{z_{12}^i}}. \quad (9)$$

⁵ Depending on the particular case under study, it may be simpler to eliminate this dependence by using one-point functions. However in this letter, following the discussion at the beginning of this section, we shall assume we do *not* have access to these quantities.

2.2. Subtracted correlators

Unfortunately the above construction cannot be applied, for instance, to the very interesting case of the Ising model which has only two non-trivial relevant operators in the spectrum. However there is a simple modification which allows this problem to be overcome.

Let us construct the differences

$$F_{jj,s}^i(t) \equiv F_{jj}^i(t) - F_{jj}^i(0) \quad (10)$$

which we shall call ‘subtracted correlators’. Since $F_{jj}^i(0)$ is a constant related to the correlator at the critical point, the subtracted correlators are almost as easily accessible from a numerical point of view as the ordinary correlators. With the help of these subtracted correlators we can construct new combinations, such as for instance

$$R_{j,s}^i \equiv \frac{\langle \phi_j \phi_j \rangle_{i,s}}{\langle \phi_j \phi_j \rangle_i} \quad (j = 1, 2) \quad (11)$$

which can be suitably combined among them or with R_{12}^i so as to eliminate the residual t dependence in the $t \rightarrow 0$ limit, thus leading to a universal combination.

2.3. QFT estimates

Let us finally address the question of the theoretical estimate of these new universal amplitude ratios. In the case of three-dimensional $O(N)$ scalar models it should be possible to obtain ϵ -expansion (or fixed dimensional) estimates by following the same lines used to describe short distance behaviour of the field correlator (see [3]). The situation is much better in two dimensions where, thanks to the progress of CFT, one knows exactly the operator content, scaling dimensions and correlators of the underlying critical QFT. Moreover, the IRS method gives an exact all order integral representation for the derivatives of Wilson’s coefficients $\widehat{\partial^n C_{jk}^\alpha / \partial g_i^n}$ in terms of CFT correlators from which accurate or even exact predictions can be obtained. With regard to VEV amplitudes, it is remarkable that the approximate technique known as truncated conformal space allows for theoretical predictions of A_α^i (see [18] and references therein) and in addition in the case of 2D integrable perturbations many A_α^i can be evaluated exactly [19]. Note that in 2D, integrable perturbations exist that are not described in terms of $O(N)$ models.

3. Example: the magnetic perturbation of the Ising model

The Ising model perturbed by the spin operator σ (in $d = 2, 3$) is the simplest non-trivial example of the previous discussion. The perturbing parameter in this case is the magnetic field h (to simplify notation we will drop in this section the perturbation label from amplitudes and ratios). The VEV of the two relevant operators are

$$\langle \sigma \rangle_h = A_\sigma h^{\frac{2\Delta_\sigma}{d-2\Delta_\sigma}} \quad \langle \epsilon \rangle_h = A_\epsilon h^{\frac{2\Delta_\epsilon}{d-2\Delta_\epsilon}}. \quad (12)$$

In the following we will concentrate on the construction of our new short distance amplitude ratios; for different amplitude ratios testing other regimes of the model, see [20, 21]. The first few terms of the short distance expansions for the correlators look like

$$\begin{aligned} F_{\sigma\sigma}(t) &\sim \widehat{C_{\sigma\sigma}^1} + A_\epsilon \widehat{C_{\sigma\sigma}^\epsilon} t^{\frac{2\Delta_\epsilon}{d-2\Delta_\epsilon}} + A_\sigma \widehat{\partial_h C_{\sigma\sigma}^\sigma} t^{1+\frac{2\Delta_\sigma}{d-2\Delta_\sigma}} \\ F_{\epsilon\epsilon}(t) &\sim \widehat{C_{\epsilon\epsilon}^1} + A_\epsilon \widehat{C_{\epsilon\epsilon}^\epsilon} t^{\frac{2\Delta_\epsilon}{d-2\Delta_\epsilon}} + A_\sigma \widehat{\partial_h C_{\epsilon\epsilon}^\sigma} t^{1+\frac{2\Delta_\sigma}{d-2\Delta_\sigma}} \\ F_{\sigma\epsilon}(t) &\sim A_\sigma \widehat{C_{\sigma\epsilon}^\sigma} t^{\frac{2\Delta_\sigma}{d-2\Delta_\sigma}} + \widehat{\partial_h C_{\sigma\epsilon}^1} t \end{aligned}$$

where $t \equiv hr^{d-2\Delta_\sigma}$, $\widehat{\partial}_h^n C_{ij}^k$ are the rescaled derivatives w.r.t. h of the Wilson coefficients. We warn the reader that the ordering of powers of t from analogous contributions is in general different in $d = 3$ and $d = 2$ due to different scale dimensions. The exact values of the first terms of short distance expansion for the case $d = 2$ are discussed in [15, 17]; here we only need to know that in $d = 2$ the coefficients $\widehat{C}_{\epsilon\epsilon}^\epsilon$ and $\widehat{\partial}_h C_{\epsilon\epsilon}^\sigma$ are exactly zero and $F_{\epsilon\epsilon,s,d=2} = \frac{1}{2}\widehat{\partial}_h^2 C_{\epsilon\epsilon}^1 t^2 + O(t^{32/15})$ (see [17]). Let us now introduce the correlator ratios as in equations (7) and (11). (Note that the leading behaviour of subtracted operators turns out to be dimension dependent.)

$$R_{\sigma\epsilon}(t) \equiv \frac{F_{\sigma\sigma}(t)F_{\epsilon\epsilon}(t)}{F_{\sigma\epsilon}^2(t)} \sim \frac{\widehat{C}_{\sigma\sigma}^1 \widehat{C}_{\epsilon\epsilon}^1}{(\widehat{C}_{\sigma\epsilon}^\sigma A_\sigma)^2} t^{-\frac{4\Delta_\sigma}{d-2\Delta_\sigma}} + \dots$$

$$R_{\sigma,s}(t) \equiv \frac{F_{\sigma\sigma,s}(t)}{F_{\sigma\sigma}(t)} \sim A_\epsilon \frac{\widehat{C}_{\sigma\sigma}^\epsilon}{\widehat{C}_{\sigma\sigma}^1} t^{\frac{2\Delta_\epsilon}{d-2\Delta_\sigma}} + \dots$$

$$R_{\epsilon,s,d=2}(t) \equiv \frac{F_{\epsilon\epsilon,s}(t)}{F_{\epsilon\epsilon}(t)} \sim \frac{1}{2} \frac{\widehat{\partial}_h^2 C_{\epsilon\epsilon}^1}{\widehat{C}_{\epsilon\epsilon}^1} t^2 + \dots$$

$$R_{\epsilon,s,d=3}(t) \equiv \frac{F_{\epsilon\epsilon,s}(t)}{F_{\epsilon\epsilon}(t)} \sim A_\epsilon \frac{\widehat{C}_{\epsilon\epsilon}^\epsilon}{\widehat{C}_{\epsilon\epsilon}^1} t^{\frac{2\Delta_\epsilon}{3-2\Delta_\sigma}} + \dots$$

It is easy to see that the simplest combinations of these ratios which in the $t \rightarrow 0$ limit lead to universal quantities are

$$Q_{\sigma\epsilon,s}(t) \equiv R_{\sigma\epsilon}^{\frac{\Delta_\epsilon}{2\Delta_\sigma}}(t) R_{\sigma,s}(t) \quad (13)$$

$$Q'_{\sigma\epsilon,s,d=2}(t) \equiv R_{\epsilon,s,d=2}^{-\frac{4}{15}}(t) R_{\sigma,s}(t) \quad (14)$$

$$Q'_{\sigma\epsilon,s,d=3}(t) \equiv R_{\epsilon,s,d=3}^{-1}(t) R_{\sigma,s}(t) \quad (15)$$

$Q'_{\sigma\epsilon,s}$ is probably the best candidate in $d = 3$, while in $d = 2$ the best choice is to study $Q_{\sigma\epsilon,s}$. Let us concentrate on the $d = 2$ case, where $\frac{\Delta_\epsilon}{2\Delta_\sigma} = 4$, and the universal ratio can be written directly in terms of correlators as

$$Q_{\sigma\epsilon,s,d=2}(t) = \frac{\langle\sigma\sigma\rangle_{h,s} (\langle\sigma\sigma\rangle_h)^3 (\langle\epsilon\epsilon\rangle_h)^4}{(\langle\sigma\epsilon\rangle_h)^8}. \quad (16)$$

The ratio $Q_{\sigma\epsilon,s,d=2}(0)$ can be obtained exactly. If we choose the CFT field normalizations such that $\widehat{C}_{\sigma\sigma}^1 = \widehat{C}_{\epsilon\epsilon}^1 = 1$ we have $\widehat{C}_{\sigma\epsilon}^\sigma = \widehat{C}_{\sigma\sigma}^\epsilon = \frac{1}{2}$. Thus we find

$$Q_{\sigma\epsilon,s,d=2}(0) = 128 \frac{A_\epsilon}{A_\sigma^8}. \quad (17)$$

By using the exact values of the VEV amplitudes (see [19] and references therein)

$$A_\sigma = 1.277\,582\,27\dots \quad A_\epsilon = 2.003\,14\dots \quad (18)$$

we finally obtain $Q_{\sigma\epsilon,s,d=2}(0) = 36.125\dots$

Acknowledgments

We thank A Pelissetto and J Zinn-Justin for helpful suggestions. This work was partially supported by the European Commission TMR program HPRN-CT-2002-00325 (EUCLID). The work of (PG) is supported by the COFIN ‘Teoria dei campi, meccanica statistica e sistemi elettronici’.

References

- [1] Zinn-Justin J *Quantum Field Theory and Critical Phenomena* 4th edn (Oxford: Oxford University Press)
- [2] Aharony A, Hohenberg P C and Privman V 1991 Universal critical point amplitude relations *Phase Transitions and Critical Phenomena* vol 14 ed C Domb and J L Lebowitz (New York: Academic)
- [3] Pelissetto A and Vicari E 2002 *Phys. Rep.* **368** 549 (Preprint cond-mat/0012164)
- [4] Lassig M 1991 *Phys. Rev. Lett.* **67** 3737
- [5] Delfino G and Mussardo G 2003 Preprint hep-th/0302056
- [6] Mussardo G 2000 Preprint hep-th/0010164
- [7] Fioravanti D, Mussardo G and Simon P 2001 *Phys. Rev. E* **63** 016103 (Preprint cond-mat/0008216)
- [8] Caselle M and Hasenbusch M 2000 *Nucl. Phys. B* **579** 667 (Preprint hep-th/9911216)
- [9] Caselle M, Hasenbusch M and Provero P 1999 *Nucl. Phys. B* **556** 575 (Preprint hep-lat/9903011)
- [10] Tarko H B and Fisher M E 1975 *Phys. Rev. B* **11** 1217
- [11] Brézin E, Amit D J and Zinn-Justin J 1974 *Phys. Rev. Lett.* **32** 151
Brézin E, Le Guillou J C and Zinn-Justin J 1974 *Phys. Rev. Lett.* **32** 473
Brézin E, De Dominicis C and Zinn-Justin J 1974 *Lett. Nuovo Cimento* **9** 483–6
Brézin E, De Dominicis C and Zinn-Justin J 1974 *Lett. Nuovo Cimento* **10** 849–51
- [12] Zamolodchikov A I B 1991 *Nucl. Phys. B* **348** 619
- [13] Mikhak B and Zarkesh A M 1994 *Nucl. Phys. B* **430** 656 (Preprint hep-th/9312207)
- [14] Guida R and Magnoli N 1996 *Nucl. Phys. B* **471** 361 (Preprint hep-th/9511209)
- [15] Guida R and Magnoli N 1997 *Nucl. Phys. B* **483** 563 (Preprint hep-th/9606072)
- [16] Caselle M, Grinza P and Magnoli N 2001 *J. Phys. A: Math. Gen.* **34** 8733 (Preprint hep-th/0103263)
- [17] Caselle M, Grinza P and Magnoli N 2000 *Nucl. Phys. B* **579** 635 (Preprint hep-th/9909065)
- [18] Guida R and Magnoli N 1997 *Phys. Lett. B* **411** 127 (Preprint hep-th/9706017)
- [19] Fateev V A 1994 *Phys. Lett. B* **324** 45
Fateev V, Lukyanov S, Zamolodchikov A B and Zamolodchikov A B 1998 *Nucl. Phys. B* **516** 652 (Preprint hep-th/9709034)
- [20] Delfino G 1998 *Phys. Lett. B* **419** 291 (Preprint hep-th/9710019)
Delfino G 2001 *Phys. Lett. B* **518** 330 (erratum)
- [21] Korff C and Seaton K A 2002 *Nucl. Phys. B* **636** 435 (Preprint cond-mat/0204232)
- [22] Fateev V, Fradkin D, Lukyanov S, Zamolodchikov A B and Zamolodchikov A B 1999 *Nucl. Phys. B* **540** 587 (Preprint hep-th/9807236)